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# Photon-Photon Scattering at the Photon Linear Collider

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## Abstract

Photon-photon scattering at the Photon Linear Collider is considered. Explicit formulas for helicity amplitudes due to  $W$  boson loops are presented. It is shown that photon-photon scattering should be easily observable at PLC and separation of the  $W$  loop contribution (which dominates at high energies) will be possible at  $e^+e^-$  c.m. energy of 500 GeV or higher.

# 1. Introduction

Experiments on the observation of the processes involving photon-photon collisions are extremely difficult to perform, because they are crossed-beam experiments which require the highest intensities and the most sensitive detection equipment. For this reason at the present time the photon-photon cross sections are still of little interest to the experimental physicist. The scattering of photons by a Coulomb field (Delbrück scattering) is the only observable case at the present time (see, *e.g.*, [1], review articles [2] and references therein).

With the advent of new collider technique [3] the collision of high energy, high intensity photon beams at the Photon Linear Collider (PLC), obtained via Compton backscattering of laser beams off linac electron beams, would provide novel opportunities for such processes. Based on the  $e^+e^-$  linear collider PLC will have almost the same energy and luminosity, *i.e.* c.m. energy of 100-500 GeV and luminosity of the order of  $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$  [3].

Explicit formulas for the fermion loop contribution to  $\gamma\gamma \rightarrow \gamma\gamma$  are well known [4]. The additional charged  $W$  boson loop contribution was shown to be finite [5] but attracted little attention until recently, when the  $W$  loop contribution to the polarization tensor of photon-photon scattering was calculated [6, 7]. The resulting expressions of a tedious calculation [6] are too complicated to be implemented numerically, and even the general properties like gauge invariance and Lorentz covariance are only checked using low energy expansions. In addition, low energy expressions obtained in [7] are not applicable above the  $W$  threshold.

In Section 2 we will give explicit analytic results for the  $W$  boson loop contributions to the helicity amplitudes and their asymptotic expressions in the high and low energy limits. Section 3 will contain numerical results. Cross sections of the photon-photon scattering in monochromatic polarized  $\gamma\gamma$  collisions as well as cross sections calculated with account of photon spectrum will be given. It is shown that photon-photon scattering cross section is large enough to be observable at the PLC and even separation of the  $W$  loop contribution will be possible at large enough energy. This fact is of fundamental significance as both triple and quartic  $W$  boson vertices contribute. Finally, in Section 4, conclusions will be made.

## 2. The $\gamma\gamma \rightarrow \gamma\gamma$ helicity amplitudes

We use the reduction algorithm from Ref. [8] to express the  $\gamma\gamma \rightarrow \gamma\gamma$  polarization tensor and helicity amplitudes in a canonical form in terms of the set of basic scalar loop integrals. All calculations were done both in 't Hooft-Feynman gauge and non-linear  $R_\xi$  gauge for  $\xi = 1$  described in [9]. The Feynman diagrams contributing to the process in non-linear gauge are shown in Fig. 1. Symmetry and transversality properties of the polarization tensor were explicitly checked. The algebraic calculations were carried out using symbolic manipulation program FORM [10].

Extracting an overall factor  $e^4/(4\pi)^2 = \alpha^2$  in the definition of the helicity amplitudes  $\mathcal{M}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ , we find three independent amplitudes:

$$\begin{aligned}\mathcal{M}_{++++}(s, t, u) = & 12 - 12\left(1 + 2\frac{t}{s}\right)B(t) - 12\left(1 + 2\frac{u}{s}\right)B(u) + 24m_W^2\frac{tu}{s}D(t, u) \\ & + \frac{4}{s}\left(4s - 6m_W^2 - 3\frac{tu}{s}\right)\left(2tC(t) + 2uC(u) - tuD(t, u)\right) \\ & + 8(s - m_W^2)(s - 3m_W^2)\left(D(s, t) + D(s, u) + D(t, u)\right); \quad (2.1)\end{aligned}$$

$$\begin{aligned}\mathcal{M}_{+++-}(s, t, u) = & -12 + 24m_W^4\left(D(s, t) + D(s, u) + D(t, u)\right) \\ & + 12m_W^2stu\left(\frac{1}{s^2}D(t, u) + \frac{1}{t^2}D(s, u) + \frac{1}{u^2}D(s, t)\right) \\ & - 24m_W^2\left(\frac{1}{s} + \frac{1}{t} + \frac{1}{u}\right)\left(sC(s) + tC(t) + uC(u)\right); \quad (2.2)\end{aligned}$$

$$\mathcal{M}_{+--+}(s, t, u) = -12 + 24m_W^4\left(D(s, t) + D(s, u) + D(t, u)\right). \quad (2.3)$$

The remaining five amplitudes can be expressed in terms of the independent ones by parity and Bose symmetry

$$\begin{aligned}\mathcal{M}_{++--}(s, t, u) &= \mathcal{M}_{+++-}(s, t, u), \\ \mathcal{M}_{+-++}(s, t, u) &= \mathcal{M}_{+++-}(s, t, u), \\ \mathcal{M}_{+-+-}(s, t, u) &= \mathcal{M}_{++++}(t, s, u), \\ \mathcal{M}_{+--+}(s, t, u) &= \mathcal{M}_{++++}(u, t, s), \\ \mathcal{M}_{+---}(s, t, u) &= \mathcal{M}_{+++-}(s, t, u),\end{aligned} \quad (2.4)$$

here  $\lambda_i$  denotes the polarization of particle  $i$  and

$$s = (p_1 + p_2)^2, \quad t = (p_2 - p_3)^2, \quad u = (p_1 - p_3)^2, \quad s + t + u = 0. \quad (2.5)$$

The scalar four-point functions are given by

$$\begin{aligned}D(s, t) = & \quad (2.6) \\ \frac{1}{i\pi^2} \int \frac{d^4q}{(q^2 - m_W^2) \left((q + p_1)^2 - m_W^2\right) \left((q + p_1 + p_2)^2 - m_W^2\right) \left((q + p_4)^2 - m_W^2\right)}, \dots\end{aligned}$$

and two- and three-point functions  $B$  and  $C$  are defined by analogous expressions. The expressions for  $B$ ,  $C$  and  $D$  functions in terms of (di-) logarithms are well known [4].

The unexpected fact is that helicity amplitudes  $\mathcal{M}_{++++}$ ,  $\mathcal{M}_{++--}$  etc., describing photon-photon scattering when algebraic sum of the photon helicities is not conserved, are just  $-3/2$  times fermion loop contribution to the corresponding amplitudes for fermion mass equal to  $m_W$ . The helicity conserving amplitudes  $\mathcal{M}_{++--}$  etc. are different for  $W$  and fermion loop contributions. Moreover, we found that  $W$  loop contribution to helicity non-conserving amplitudes were just 3 times the charged scalar loop contribution. So, if we consider photons interacting with “supersymmetric” set of charged scalar, fermion and vector fields degenerate in mass (*i.e.*, if the number of bosonic degrees of freedom of charged fields is equal to the number of fermionic ones), helicity non-conserving amplitudes will add exactly to zero and only helicity conserving amplitudes will survive! It seems, this should be explained by the embedding of the Yang-Mills theory in a supersymmetric theory.

In the low energy limit  $s \ll m_W^2$  the leading terms of the expansion of helicity amplitudes are

$$\mathcal{M}_{++++} = \frac{14}{5} \frac{s^2}{m_W^4}, \quad \mathcal{M}_{++--} = \frac{1}{10} \frac{s^2 + t^2 + u^2}{m_W^4}, \quad \mathcal{M}_{+-+-} = 0, \dots, \quad (2.7)$$

and simple closed expressions are easily obtained for the differential cross section

$$\frac{d\sigma^W}{d\cos\theta} = \frac{393\alpha^4}{25600\pi} \frac{s^3}{m_W^8} (1 + \cos^2\theta)^2 \quad (2.8)$$

and total cross section

$$\sigma_{tot}^W = \frac{2751\alpha^4}{16000\pi} \frac{s^3}{m_W^8} \approx 0.055\alpha^4 \frac{s^3}{m_W^8}, \quad (2.9)$$

in precise agreement with the low energy expansion result of [7].

The ratio of the electron and  $W$  loop induced cross sections at  $\sqrt{s} \approx 3m_e$ , where  $\sigma^e$  has a maximum of  $1.6 \mu\text{b}$  [4], is extremely small  $\sigma^W/\sigma^e \approx 115(m_e/m_W)^8 \approx 10^{-40}$ . That is one of the reasons why  $W$  loop contribution has been ignored so far.

In the high energy limit with  $u$  held finite and large,  $s \gg -u \gg m_W^2$ , which corresponds to photon scattering at small angle,  $\theta \sim (-u/s)^{1/2}$ , only two helicity conserving amplitudes survive, which are imaginary and proportional to  $s$

$$\mathcal{M}_{++++} = \mathcal{M}_{--++} = -16\pi i \frac{s}{u} \log(-u/m_W^2), \quad (2.10)$$

while leading fermion loop contributions to helicity amplitudes grow only as logarithm squared [11, 4]

$$\mathcal{M}_{++++}^f = \mathcal{M}_{--++}^f = -4 \log^2(-s/u). \quad (2.11)$$

It means that at high energies the  $W$  loop contribution will not only be non-negligible, but will even dominate in the total cross section of photon-photon scattering. The fact that  $W$  loop contribution gives rising amplitudes and non-decreasing total cross

section in the high energy limit is a consequence of the  $u$ ,  $t$ -channel vector particle exchanges [12]. Such behaviour can be qualitatively understood by considering amplitudes for forward (backward) scattering that can easily be calculated via optical theorem

$$\alpha^2 \mathcal{I}m \mathcal{M}_{++++(+--+)}^W(u=0) = s \sigma_{\gamma\gamma \rightarrow W^+W^-}^{++(+ -)} = 8\pi\alpha^2 s / m_W^2. \quad (2.12)$$

### 3. Cross sections

Fig. 2 presents polarized cross sections for photon-photon scattering in monochromatic  $\gamma\gamma$  collisions summed over final photon helicities. We consider here the extreme cases of  $\lambda_1\lambda_2 = \pm 1$ , *i.e.* full circular polarization for the incoming photons. The cross section is given by

$$\frac{d\sigma_{\lambda_1, \lambda_2}(s)}{d\cos\theta} = \sum_{\lambda_3, \lambda_4} \frac{\alpha^4}{32\pi s} \left| \mathcal{M}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} \right|^2, \quad (3.1)$$

where the integration over  $\cos\theta$  should be done from 0 to 1. The parameters  $\alpha = 1/128$ ,  $m_W = 80.22$  GeV and  $m_t = 120$  GeV have been used throughout the paper. We present total cross sections as well as separate contributions coming from  $W$ -boson loop and fermion loop. To get an idea of an observable cross section, we restrict the photon scattering angle,  $|\cos\theta| < \cos 30^\circ$ . As expected, below  $W$  threshold the fermion loop contribution is dominating, while  $W$  loop dominates at photon-photon collision energies above  $200 \div 250$  GeV.

In Fig. 3 we show the total cross sections of the photon-photon scattering calculated with account of the photon spectrum [3] as a function of the  $e^+e^-$  c.m. energy for unpolarized initial electron and laser beams. The large cross section originating from fermion loop contribution is due to a low energy tail of the photon distribution function. *E.g.*, for  $e^+e^-$  c.m. energy below 200 GeV the cross section is of the order of 100 fb, which corresponds to about a thousand of events for the integrated luminosity of  $10 \text{ fb}^{-1}$ . To be prudent we also calculated the background from resolved photon contribution  $q\bar{q} \rightarrow \gamma\gamma$ . The invariant mass of two photons was taken to be higher than 20 GeV. We used the parametrization of photonic parton distributions [13]. The resolved photon contribution is sizable, but it also comes from low energy photon-photon pair production.

Fig. 4 shows that if one cuts off photon pairs with low invariant mass it will be possible to separate the  $W$  loop contribution. For  $M_{\gamma\gamma} > 250$  GeV at  $e^+e^-$  c.m. energy of 500 GeV the  $W$  contribution to cross section is dominating,  $\sigma_W \approx 5$  fb (which corresponds to 50 events for  $\int d\mathcal{L} = 10 \text{ fb}^{-1}$ ), while the fermion contribution is negligible,  $\sigma_f \approx 0.2$  fb. And the resolved photon contribution is completely negligible for  $M_{\gamma\gamma} > 100$  GeV.

Future detectors possibilities to detect two photon production reactions were carefully studied for rare two-photon decay of the Higgs particle [14]. The achievable precision of the photon angular measurements and two photon invariant mass resolutions

were shown to be 1 mrad and 1-3 GeV, respectively [14]. In our case much simpler electromagnetic calorimeter would be enough to study photon-photon scattering reaction. In practice one of the major sources of background is the fragmentation of jet into a leading neutral meson ( $\pi^0$ ,  $\eta^0$ , or  $K_L^0$ ). Fortunately, we can use the value of current rejection factor of  $5 \times 10^{-4}$  obtained by CDF in their single photon analysis [15], which is defined as the ratio of background in their inclusive single photon cross section to the total inclusive jet cross section. The jet cross section in  $\gamma\gamma$  collisions for jet-jet invariant mass above 100 GeV in the  $p_t$  region above 30 GeV can be estimated to be below 100 pb. Multiplying this cross section by the rejection factor squared we obtain that this background is really negligible.

For the case of polarized electron and laser photon beams the low energy photon tail can be significantly suppressed by varying the value of the conversion distance  $z$  (distance from the conversion point – where the laser pulse intersects the electron beam – to the interaction point) [3]. Plotted in Fig. 5 are the total cross sections as a function of the  $e^+e^-$  c.m. energy for polarized electron and laser beams and a conversion distance  $z = 2.5$  cm. The  $W$ -loop contribution clearly dominates over fermion loop contribution without any  $\gamma\gamma$  invariant mass cut, while the value of the cross section due to  $W$ -loop is only 20% smaller than that for zero conversion distance (Fig. 3). The resolved photon contribution is about 2 fb at  $e^+e^-$  c.m. energy of 500 GeV and can be further reduced by photon-photon invariant mass cut (Fig. 4).

## 4. Conclusions

In conclusion, photon-photon scattering should easily be observable at the PLC. Fermion loop contribution dominates below the  $W$  threshold, but at photon-photon collision energies above  $200 \div 250$  GeV the  $W$  loop contribution is dominating. For  $e^+e^-$  c.m. energy of 500 GeV or higher it will be possible to separate the  $W$  contribution. The observation of the photon-photon scattering due to the  $W$  loop will have fundamental significance, because this process is a pure one-loop effect of the Standard Model as a renormalizable nonabelian gauge theory. In principle, this reaction can be used to probe the anomalous triple and quartic  $W$  boson vertices.

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## Figure captions

Fig. 1. Feynman graphs contributing to the process  $\gamma\gamma \rightarrow \gamma\gamma$ . Notations are the following: photon – wavy line,  $W$  boson – zigzag line, charged NG scalar – solid line, FP ghost – dashed line.

Fig. 2. Total cross section of photon-photon scattering in monochromatic photon-photon collisions versus  $\gamma\gamma$  c.m. energy for different helicities of the incoming photons. Total cross section (solid line) as well as  $W$  boson loop contribution (dashed line) and fermion loop contribution (dotted line) are shown.

Fig. 3. Cross section of photon-photon scattering in  $\gamma\gamma$  collisions versus c.m. energy of the  $e^+e^-$  collisions computed taking into account photon spectrum of the backscattered laser beams. Dash-dotted line shows the resolved photon contribution.

Fig. 4. The invariant mass,  $M_{\gamma\gamma} > M_{cut}$ , distribution versus  $M_{cut}$  for  $\gamma\gamma \rightarrow \gamma\gamma$  at  $\sqrt{s_{e^+e^-}} = 500$  GeV for polarized electron and laser beams. Notations are the same as in Fig. 3.

Fig. 5. Cross section of photon-photon scattering in  $\gamma\gamma$  collisions versus c.m. energy of the  $e^+e^-$  collisions for polarized electron and laser beams. The electron beam spotsize is 100 nm and the conversion distance is  $z = 2.5$  cm.



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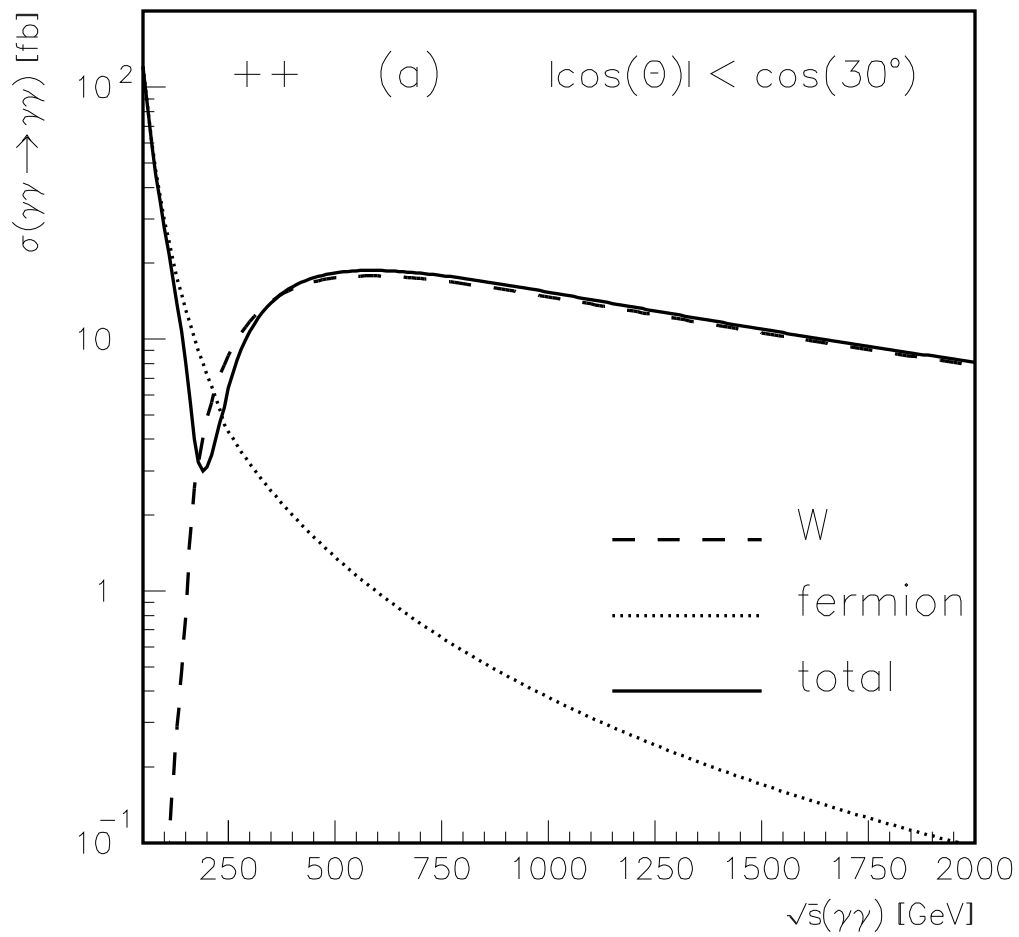
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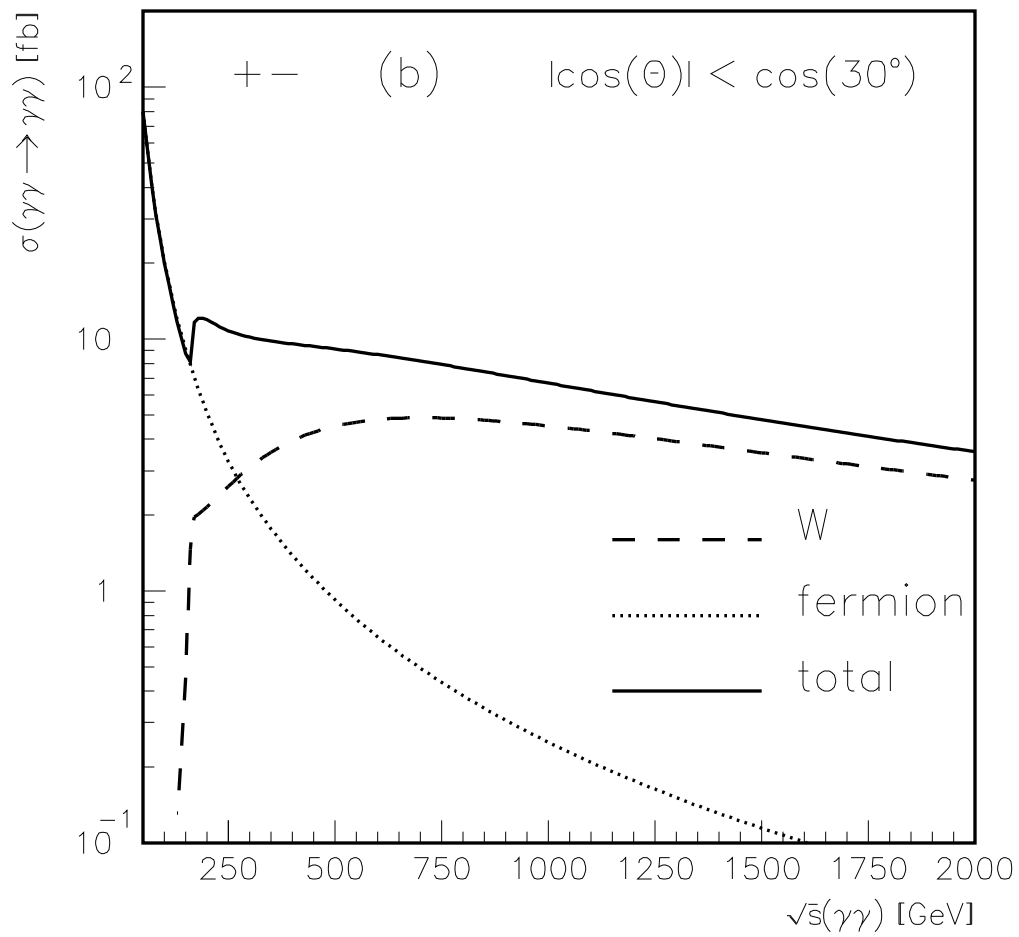
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